An edge pedestal model based on transport and atomic physics

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A model is presented for the calculation of the characteristic scale lengths from transport considerations in the edge pedestal region of high confinement (*H*-mode) plasmas. The model is based on the requirements of heat and particle removal through the edge. Atomic physics effects on edge density and temperature gradient scale lengths are taken into account. An empirical fit for the width of the edge pedestal transport barrier is employed. Model problem calculations indicate that the model predicts the magnitudes and some trends of characteristic gradient scale lengths observed in current experiments. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1388175]

I. INTRODUCTION

The steep gradient region in the edge of *H*-mode (high confinement mode) tokamak plasmas, the so-called "edge transport barrier," plays an important role in many aspects of tokamak physics and is a topic of active experimental and theoretical investigation. The maximum pressure gradient in this edge transport barrier has long been thought to be limited by ideal magnetohydrodynamic (MHD) ballooning modes, but recent experimental results[1,2] indicate that the pressure gradient in this region can exceed the nominal first ideal stability boundary (that is, the ideal stability boundary extrapolated from the core, not taking into account the effects of geometry, shear, and current in the edge) for ballooning modes, which has important implications for the performance of tokamaks.

Two explanations have been proposed for these pressure gradients which exceed the nominal ideal first stability limit. One suggestion[1–3] is that the edge pressure gradient drives a bootstrap current that affects the ballooning stability limit and can even entirely remove the ballooning mode stability limit by allowing access to the second stability regime. An alternate explanation based on the stabilization of ballooning modes by diamagnetic effects has been proposed,[4] employing the three-dimensional Braginskii equations and accounting for the localization of the pressure gradient and for ion diamagnetic effects. This analysis indicates that ballooning modes become stable and the maximum pressure gradient is determined by a stability limit on the pedestal β, which can be cast in the form of the ballooning mode limit with a multiplicative enhancement factor.

Although the MHD constraints may limit the maximum pressure gradient, the individual density and temperature gradients in the edge transport barrier must be consistent with the particle and heat fluxes that are flowing through the edge transport barrier and with atomic physics effects on these fluxes. This observation suggests that a pedestal model in which the individual density and temperature gradients are determined from particle and heat flux requirements, but which are constrained by a maximum allowable pressure gradient determined from MHD theory, may serve useful predictive and interpretative purposes. As an initial step toward such a model, we have developed a pedestal model in which the gradient scale lengths are calculated from transport considerations, taking into account atomic physics effects, and which uses an empirical fit for the edge transport barrier width. The purpose of this paper is to present and investigate such a pedestal model.

II. PEDESTAL MODEL

A. Transport constraints on gradient scale lengths

The ion flux passing through the transport barrier satisfies

$$\frac{d\Gamma_{\perp}}{dr} = n n_0 \langle \sigma v \rangle_{\text{ion}} \equiv n \nu_{\text{ion}}, \tag{1}$$

where n_0 is the neutral atom density and $\langle \sigma v \rangle_{\text{ion}}$ is the specific ion-electron ionization rate averaged over the energy distributions of both species. Integrating this equation from the top of the pedestal (ped) outward to the seperatrix (sep) yields

$$\Gamma_{\perp}^{\text{sep}} - \Gamma_{\perp}^{\text{ped}} = \int_{\Delta_{\text{TB}}} n \nu_{\text{ion}}\, dr \equiv n_{\text{TB}} \nu_{\text{ion}}^{\text{TB}} \Delta_{\text{TB}}, \tag{2}$$

where $\Gamma_{\perp}^{\text{sep}}$ is the net outward ion current crossing the separatrix from the plasma edge into the scrape-off layer, $\Gamma_{\perp}^{\text{ped}}$ is the net outward ion current from the core plasma into the transport barrier at the top of the pedestal, and Δ_{TB} is the width of the region from the pedestal to the separatrix. Note that the ion current is not constant across the transport barrier but increases radially outward because of ionization of neutrals.

In order to define an average density gradient in the transport barrier, we define an average value of the net outward ion current

$$\Gamma_{\perp}^{\text{av}} \equiv \tfrac{1}{2}(\Gamma_{\perp}^{\text{sep}} + \Gamma_{\perp}^{\text{ped}}), \tag{3}$$

which we then relate to the general form for the ion current

$$\Gamma_{\perp}^{\text{av}} = -D\frac{dn}{dr} + n_{\text{TB}} v_p = n_{\text{TB}}(D L_n^{-1} + v_p), \tag{4}$$

where D is the diffusion coefficient and v_p is the "pinch velocity." We may eliminate either $\Gamma_\perp^{\mathrm{sep}}$ or $\Gamma_\perp^{\mathrm{ped}}$ by using Eqs. (2) and (3). Because we will determine $\Gamma_\perp^{\mathrm{sep}}$ from a particle balance on the entire region inside the separatrix (see Sec. III), we elect to eliminate $\Gamma_\perp^{\mathrm{ped}}$ to obtain an expression for the density gradient scale length in the edge transport barrier

$$L_n=\frac{D}{\Gamma_\perp^{\mathrm{sep}}/n_{\mathrm{TB}}-\frac{1}{2}\nu_{\mathrm{ion}}^{\mathrm{TB}}\Delta_{\mathrm{TB}}-v_p}. \tag{5}$$

Assuming that the ions and electrons cross the transport barrier in a time short compared to the equilibration time, the net outward electron and ion heat fluxes in the transport barrier satisfy

$$\begin{aligned}\frac{dQ_{\perp e}}{dr}&=-nn_0\langle\sigma v\rangle_{\mathrm{ion}}E_{\mathrm{ion}}-nn_zL_z\\ &\equiv-n\nu_{\mathrm{ion}}E_{\mathrm{ion}}-nn_zL_z,\end{aligned} \tag{6}$$

and

$$\frac{dQ_{\perp i}}{dr}=-nn_0^c\langle\sigma v\rangle_{\mathrm{at}}\frac{3}{2}T\equiv-n\nu_{\mathrm{at}}^c\frac{3}{2}T_i, \tag{7}$$

where E_{ion} is the ionization energy, n_z and L_z are impurity density and radiation emissivity, n_0^c is the uncollided (cold) neutral density in the transport barrier, and $\langle\sigma v\rangle_{\mathrm{at}}$ is the specific elastic scattering plus charge exchange reaction rate of previously uncollided (in the transport barrier) neutrals. These equations may be integrated across the transport barrier to obtain

$$\begin{aligned}Q_{\perp e}^{\mathrm{sep}}-Q_{\perp e}^{\mathrm{ped}}&=-\int_{\Delta_{\mathrm{TB}}}n\nu_{\mathrm{ion}}E_{\mathrm{ion}}\,dr-\int_{\Delta_{\mathrm{TB}}}nn_zL_z\,dr\\ &\equiv-n_{\mathrm{TB}}\nu_{\mathrm{ion}}^{\mathrm{TB}}E_{\mathrm{ion}}\Delta_{\mathrm{TB}}-n_{\mathrm{TB}}(n_zL_z)_{\mathrm{TB}}\Delta_{\mathrm{TB}}\end{aligned} \tag{8}$$

and

$$Q_{\perp i}^{\mathrm{sep}}-Q_{\perp i}^{\mathrm{ped}}=-\int_{\Delta_{\mathrm{TB}}}n\nu_{\mathrm{at}}^c\frac{3}{2}T_i\,dr\equiv-n_{\mathrm{TB}}\nu_{\mathrm{at}}^{c\mathrm{TB}}\frac{3}{2}T_i^{\mathrm{TB}}\Delta_{\mathrm{TB}}. \tag{9}$$

Proceeding as above, and equating the average heat flux to the standard form

$$Q_\perp^{\mathrm{av}}=-\chi n\frac{dT}{dr}+\frac{3}{2}T\Gamma_\perp=\chi nTL_T^{-1}+\frac{3}{2}T\Gamma_\perp. \tag{10}$$

leads to expressions for average electron and ion temperature gradient scale lengths in the transport barrier

$$L_{Te}=\frac{\chi_e^{\mathrm{TB}}}{\left[\left(\frac{Q_{\perp e}^{\mathrm{sep}}}{n_{\mathrm{TB}}T_e^{\mathrm{TB}}}-\frac{3}{2}\frac{\Gamma_\perp^{\mathrm{sep}}}{n_{\mathrm{TB}}}\right)+\frac{1}{2}\Delta_{\mathrm{TB}}\left(\frac{(n_zL_z)_{\mathrm{TB}}}{T_e^{\mathrm{TB}}}+\nu_{\mathrm{ion}}^{\mathrm{TB}}\left(\frac{E_{\mathrm{ion}}}{T_e^{\mathrm{TB}}}+\frac{3}{2}\right)\right)\right]} \tag{11}$$

and

$$L_{Ti}=\frac{\chi_i^{\mathrm{TB}}}{\left[\left(\frac{Q_{\perp i}^{\mathrm{sep}}}{n_{\mathrm{TB}}T_i^{\mathrm{TB}}}-\frac{3}{2}\frac{\Gamma_\perp^{\mathrm{sep}}}{n_{\mathrm{TB}}}\right)+\frac{1}{2}\Delta_{\mathrm{TB}}\frac{3}{2}(\nu_{\mathrm{at}}^{c,\mathrm{TB}}+\nu_{\mathrm{ion}}^{\mathrm{TB}})\right]}, \tag{12}$$

where the χ's are average thermal diffusivities for ions and electrons in the transport barrier.

As Eqs. (5), (11), and (12) make clear, the gradient scale lengths in the transport barrier depend on the particle and heat fluxes flowing through the transport barrier (which must be determined by the particle and heat balances on the core plasma), on the transport coefficients in the transport barrier, and on the atomic physics particle and heat sources and sinks in the transport barrier. Thus, these gradient scale lengths cannot be determined just on the basis of a local model for the pedestal, but must take into account also the core plasma balance and the fueling and recycling neutrals in the edge transport barrier.

B. Critical pressure gradient constraint

The gradient scale lengths determined from transport considerations are constrained by magnetohydrodynamic (MHD) stability requirements. This constraint is conventionally written in the form

$$-\left(\frac{dp}{dr}\right)_{\mathrm{crit}}=\frac{\alpha_c\left(\frac{B^2}{2\mu_0}\right)}{q_{95}^2R}, \tag{13}$$

where B is the toroidal field, R is the major radius, q_{95} is the safety factor at the 95% flux surface, and α_c is in general a function of magnetic shear and plasma geometry. The nominal ideal ballooning mode value of α_c is of order unity in the absence of second stability access. Access to second stability increases α_c somewhat, to the point at which lower toroidal mode number (n) modes, which do not have access to second stability, become unstable.[5] The β-limit result of Ref. 4 corresponds to[2]

$$\alpha_c=\frac{\rho_i}{\Delta_{\mathrm{TB}}}\left(\frac{2}{1+T_i/T_e}\frac{R}{\Delta_{\mathrm{TB}}}\right)^{1/2}, \tag{14}$$

where ρ_i is the ion gyroradius and Δ_{TB} is the width of the steep gradient region in the edge (the edge transport barrier).

The MHD pressure gradient, or β, constraint and the transport constraints discussed previously must interact in some manner to determine the width of the transport barrier, Δ_{TB}. Other phenomena may also be involved in the determination of Δ_{TB}. Although there are several theories for Δ_{TB}, none of them are in particularly good agreement with experiment.[6] Since our principal purpose in this paper is to examine the possibility that gradient scale lengths in the edge transport barrier are determined by transport constraints, and since we will use a DIII-D model problem calculation for this purpose, we will use an empirical fit[2] to the DIII-D data to evaluate Δ_{TB},

$$\Delta_{TB}=C_0R\left(\frac{n_{ped}T_{ped}}{B_\theta^2/2\mu_0}\right)^{0.4}. \tag{15}$$

Here the subscript ped refers to the value at the pedestal at the top of the steep gradient region, B_θ is the poloidal magnetic field, and $C_0=0.02$ is a constant that we have found to provide a reasonable fit to a limited number of DIII-D shots that have been examined for this purpose.

Noting that the total pressure gradient may be written

$$-\left(\frac{dp}{dr}\right)=-p\frac{1}{p}\left(\frac{dp}{dr}\right)=-p\frac{1}{n}\frac{dn}{dr}-p_e\frac{1}{T_e}\frac{dT_e}{dr}-p_i\frac{1}{T_i}\frac{dT_i}{dr}$$
$$\equiv pL_n^{-1}+p_eL_{T_e}^{-1}+p_iL_{T_i}^{-1}, \tag{16}$$

in terms of the density and temperature gradient scale lengths, $L_{n,T}$, the pressure gradient constraint may be written

$$-\frac{1}{p}\left(\frac{dp}{dr}\right)_{crit}\equiv L_{MHD}^{-1}\geqslant L_n^{-1}+\frac{L_{T_e}^{-1}}{1+T_i/T_e}+\frac{L_{T_i}^{-1}}{1+T_e/T_i}$$
$$\equiv L_*^{-1}. \tag{17}$$

III. MODEL PROBLEM CALCULATIONS

The pedestal model described above has been coupled with a model for the core plasma particle and power balances, a model for the scrape-off layer (SOL) and divertor plasma particle, momentum and power balances, and a model for the transport of fueling and recycling neutrals, all of which have been developed for and checked against analyses of DIII-D (see the Appendix).

We have performed a number of calculations to investigate the pedestal model of Sec. II on a DIII-D model problem ($R=1.75$ m, $a=0.6$ m, $\kappa=1.74$, $\delta=0.74$, $B=2.0$ T, $I=1.03$ MA, $q_{95}=4.4$, $H_{89F}=1.8$, upper single null [USN] divertor). A range of low auxiliary power, gas fueled conditions were simulated. The particle and heat fluxes crossing the separatrix were calculated from particle and power balances on the entire plasma, and the fueling and recycling neutrals in the transport barrier were calculated directly. The gradient scale lengths in the transport barrier were calculated from the transport model of Eqs. (5), (11), and (12).

Then the pedestal density and temperature were evaluated from the gradient scale lengths, the width of the transport barrier, and the values of density and temperatures calculated at the separatrix from a SOL-divertor plasma calculation. The model used for the core and SOL-divertor plasma calculations in this paper did not distinguish between ion and electron temperatures, so it was necessary to calculate ion and electron temperatures at the separatrix and pedestal from the available average separatrix temperature T_{SOL},

$$T_{SOL}=\tfrac{1}{2}(T_e^{SOL}+T_i^{SOL}) \tag{18}$$

and the relationships

$$T_{i,e}^{ped}=T_{i,e}^{SOL}e^{\Delta_{TB}/L_{T_{i,e}}}. \tag{19}$$

These equations can be solved for the electron and ion pedestal temperatures

$$T_e^{ped}=\frac{2T_{SOL}}{\left(e^{-\Delta_{TB}/L_{T_e}}+\left(\frac{T_i^{ped}}{T_e^{ped}}\right)e^{-\Delta_{TB}/L_{T_i}}\right)} \tag{20}$$

and

$$T_i^{ped}=\left(\frac{T_i^{ped}}{T_e^{ped}}\right)T_e^{ped} \tag{21}$$

in terms of the ratio $C_2\equiv(T_i^{ped}/T_e^{ped})$, which is typically in the range 1–2. The ion and electron separatrix temperatures

TABLE I. Effect of transport coefficient on the characteristic scale lengths in the edge transport barrier ($R=1.76$ m, $a=0.6$ m, $\kappa=1.76$, $\delta=0.22$, $B=2.0$ T, $I=1.0$ MA, $q_{95}=4.8$, $H_{89P}=2.0$, $H_N/H_{89}=0.5$, USN divertor, $p_{nbi}=2.0$ MW, $S=3.0\times10^{21}$ s, $v_p=0.0$, $C_0=0.02$, $C_2=1.5$).

$\chi_i=\chi_e$ (m^2/s)	L_n (cm)	L_{T_e} (cm)	L_{T_i} (cm)	L_{RD} (cm)	Δ_{TB} (cm)	BI
$D=1/3\chi$						
0.2	1.2	1.3	2.2	2.0	1.3	3.64
0.3	1.3	1.3	2.1	1.1	1.1	1.86
0.4	1.5	1.3	2.2	0.7	0.9	1.07
0.5	1.8	1.3	2.2	0.6	0.9	0.82
$D=\chi$						
0.3	2.8	1.0	1.7	0.6	0.9	0.78
0.4	3.6	1.1	1.8	0.4	0.8	0.55
0.5	4.3	1.1	1.9	0.4	0.8	0.43

TABLE II. Effect of inward pinch velocity on the characteristic scale lengths in the edge transport barrier ($R = 1.76$ m, $a = 0.6$ m, $\kappa = 1.76$, $\delta = 0.22$, $B = 2.0$ T, $I = 1.0$ MA, $q_{95} = 4.8$, $H_{89P} = 2.0$, $H_N/H_{89} = 0.5$, USN divertor, $P_{\mathrm{nbi}} = 2.0$ MW, $S = 3.0\times 10^{21}$ s, $\chi_i = \chi_e = 0.5$ m^2/s, $D = 1/3\chi$, $C_0 = 0.02$, $C_2 = 1.5$, $Q_{\perp e}/Q_{\perp} = 0.5$).

v_p (m/s)	L_n (cm)	L_{T_e} (cm)	L_{T_i} (cm)	L_{RD} (cm)	Δ_{TB} (cm)	BI
0.0	1.8	1.3	2.2	0.6	0.9	0.82
−2.5	1.5	1.4	2.3	0.7	0.9	1.02
−5.0	1.3	1.5	2.5	0.8	1.0	1.28
−7.5	1.1	1.6	2.6	0.9	1.0	1.62
−10.0	1.0	1.8	2.9	1.1	1.1	2.04

may then be calculated from Eq. (19), and the average pedestal temperature for use in the core plasma calculation may be constructed from

$$T_{\mathrm{ped}} = \tfrac{1}{2}(T_e^{\mathrm{ped}} + T_i^{\mathrm{ped}}). \tag{22}$$

All of these interactive calculations were iterated to consistency, so that any change in pedestal model parameter affected the core plasma, SOL-divertor plasma, and neutral transport calculations, which in turn affected particle and heat fluxes, separatrix densities and temperatures, neutral concentrations in the transport barrier, etc., that enter into the pedestal calculation.

Since the core and SOL plasma models did not distinguish between ion and electron temperatures, it was necessary to make certain assumptions: that the power flux crossing the separatrix was evenly distributed between the ions and electrons, that the ion and electron thermal diffusivities in the transport barrier were equal, and that the ion temperature was 1.5 times the electron temperature at the top of pedestal.

At the present state of development, the MHD pressure gradient constraint does not enter directly into the calculation model though it may be implicitly included in the empirical fit used for Δ_{TB}. We plan in the future to replace the empirical fit with a theoretical model for Δ_{TB} based on the MHD pressure constraint. For now, we will compare the pressure scale length L_* calculated from transport considerations with the limiting pressure gradient scale length $L_{\mathrm{RD}} \equiv L_{\mathrm{MHD}}$ calculated with the Rogers–Drake model. For this purpose, we define a "beta index"

$$\mathrm{BI} \equiv \frac{L_{\mathrm{RD}}}{L_*} = \left(L_n^{-1} + \frac{L^{1}_{T_e}}{1 + T_i/T_e} + \frac{L^{1}_{T_i}}{1 + T_e/T_i} \right) \Big/ L_{\mathrm{RD}}^{-1}. \tag{23}$$

$\mathrm{BI} < 1$ indicates that the pressure gradient calculated from transport considerations is less than the critical pressure gradient predicted by the Rogers–Drake model.

The calculated density and temperature gradient scale lengths, the limiting MHD pressure gradient scale length L_{RD}, the transport barrier width Δ_{TB}, and the beta index are tabulated in Table I for calculations with $P_{\mathrm{nbi}} = 2.0$ MW, a fueling source $S = 3.0\times 10^{21}$ s, $v_p = 0.0$, $C_0 = 0.02$, and $C_2 = 1.5$. The calculation was repeated for several values of $\chi_i = \chi_e$ and for $D = 1/3\chi$ and for $D = \chi$. We note that $L_{T_i} > L_{T_e}$ as commonly observed in DIII-D,[1] that $L_T < L_n$ for $D = \chi$ as commonly observed in ASDEX-Upgrade,[7] that L_{RD} and Δ_{TB} are similar in magnitude, and that all of these quantities are of the magnitude observed in these experiments. The effects of increasing $\chi_i = \chi_e \sim D$ are to increase the density gradient scale length significantly but increase the temperature gradient scale lengths only slightly, to increase the critical pressure gradient (hence to decrease L_{RD}), and to decrease the beta index. The choice $D = 1/3\chi$ results in somewhat smaller density gradient scale lengths and somewhat larger temperature gradient scale lengths than does the choice $D = \chi$. Solutions with $\mathrm{BI} > 1$ would not be allowed by MHD stability constraints if the β limit of Ref. 4 is governing, but possibly would be allowed if the second-stability regime suggestion of Ref. 1 is governing.

The sensitivity of the results to the value of the inward pinch velocity is illustrated in Table II. These calculations were made for the same parameters mentioned previously but now with $\chi_i = \chi_e = 0.5\,\mathrm{m}^2/\mathrm{s}$ and $D = 1/3\chi$. The effect of increasing the inward pinch velocity is to reduce L_n and to increase L_{T_i}, L_{T_e}, and L_{RD}, which eventually leads to violation of the MHD stability condition ($\mathrm{BI} > 1$).

Sensitivity of the model to the choice of the constants C_0 and C_2 is illustrated in Table III. The width of the transport barrier, given by Eq. (15), scales linearly with C_0, and the quantity α_c and the critical pressure gradient scale as $\Delta_{\mathrm{TB}}^{3/2}$ (hence L_{RD} scales as $\Delta_{\mathrm{TB}}^{3/2}$). An increase in C_0 (Δ_{TB}) also indirectly causes an increase in the density and temperature gradient scale lengths, which partially offsets the increase in

TABLE III. Sensitivity of the characteristic scale lengths in the edge transport barrier to model parameters ($R = 1.76$ m, $a = 0.6$ m, $\kappa = 1.76$, $\delta = 0.22$, $B = 2.0$ T, $I = 1.0$ MA, $q_{95} = 4.8$, $H_{89P} = 2.0$, $H_N/H_{89} = 0.5$, USN divertor, $P_{\mathrm{nbi}} = 2.0$ MW, $S = 3.0\times 10^{21}$ s, $v_p = 0.0$, $\chi_i = \chi_e = 0.5$ m^2/s, $D = 1/3\chi$, $v_p = 0.0$, $Q_{\perp e}/Q_{\perp} = 0.5$).

C_0	C_2	α_c	L_n (cm)	L_{T_e} (cm)	L_{T_i} (cm)	L_{RD} (cm)	Δ_{TB} (cm)	BI
0.02	1.5	4.31	1.8	1.3	2.2	0.6	0.9	0.82
0.03	1.5	2.60	2.1	1.9	3.1	1.4	1.5	1.54
0.02	2.0	4.56	1.8	1.2	3.1	0.6	0.9	0.81

TABLE IV. Effect of gas fueling rate on the characteristic scale lengths in the edge transport barrier ($R=1.76$ m, $a=0.6$ m, $\kappa=1.76$, $\delta=0.22$, $B=2.0$ T, $I=1.0$ MA, $q_{95}=4.8$, $H_{89P}=2.0$, $H_N/H_{89}=0.5$, USN divertor, $P_{\mathrm{nbi}}=2.0$ MW, $v_p=0.0$, $\chi_i=\chi_e=0.5$ m^2/s, $D=1/3\chi$, $C_0=0.02$, $C_2=1.5$, $Q_{\perp e}/Q_{\perp}=0.5$).

S (10^{21}/s)	$Q_\perp$ (10^4 W/s)	Γ (m^2 s)	L_n (cm)	L_{T_e} (cm)	L_{T_i} (cm)	L_{RD} (cm)	Δ_{TB} (cm)	BI
3.0	3.3	1.2	1.8	1.3	2.2	0.6	0.9	0.82
4.0	3.2	1.4	1.7	1.4	2.4	0.6	0.9	0.87
6.0	3.2	1.7	1.6	1.5	2.7	0.7	0.9	0.95
8.0	3.1	2.0	1.5	1.7	3.0	0.7	0.9	1.01
10.0	3.0	2.3	1.4	1.8	3.4	0.8	0.9	1.05
12.0	2.9	2.6	1.4	1.9	3.8	0.8	0.9	1.08
14.0	2.8	3.0	1.3	2.0	4.3	0.8	0.9	1.10

the beta index caused by the decrease in critical pressure gradient produced by an increase in C_0. The ratio L_{T_i}/L_{T_e} varies as the parameter $C_2 \equiv (T_i^{\mathrm{ped}}/T_e^{\mathrm{ped}})$. [These parameters (perhaps functions of other variables) must at this point be determined empirically for each experiment.]

A series of calculations with different values of the gas fueling rate is summarized in Table IV. Increasing the gas fueling rate increases $\Gamma_\perp$, which in turn decreases the density gradient scale length, and decreases $Q_\perp$, which increases the temperature gradient scale lengths. The increase in temperature gradient scale length with increased gas fueling rate is consistent with DIII-D data.[8]

A series of calculations at different heating powers is summarized in Table V. Increasing the heating power increases the pedestal pressure, which increases the transport barrier width Δ_{TB} and the MHD gradient scale length and beta index. The increase in atomic physics reaction rates in the transport barrier with increased heating, hence increased temperature in the transport barrier, acts to increase the density gradient scale length and to offset the effect of an increase in $Q_\perp$ on the temperature gradient scale lengths.

As emphasized in Sec. II, the characteristic scale lengths in the edge pedestal depend not only on the local parameters but also on the heat and particle fluxes flowing through the edge and on the neutral influx into the plasma edge, and hence on the overall solution for the plasma and neutral parameters. Various parameters characterizing the overall solution are presented in Table VI for a representative case considered in this analysis.

IV. SUMMARY AND CONCLUSIONS

A "pedestal model" for the calculation of characteristic scale lengths in the steep-gradient region in the edge of H-mode plasmas is proposed. The density gradient scale length is calculated from the net particle current passing through the edge, and the temperature gradient scale lengths are calculated from the net heat fluxes passing through the edge. An empirical fit for the edge pedestal transport barrier width is employed. Model problem calculations reproduce the magnitude and several trends of the characteristic scale lengths observed experimentally.

Although the gradient scale length calculations based on transport and edge atomic physics considerations are generally consistent with experimental observation, the present model depends on a number of empirical parameters, foremost among which are the edge transport parameters (χ, D, v_p), the width of the edge transport barrier, and the core particle and energy confinement times. The next step in the development (evaluation) of the pedestal model of this paper should be a detailed comparison of calculated gradient scale lengths with experimental values for specific devices, using the respective empirical parameters for each device.

However, the true utility of such a model will only be realized when it becomes more predictive and less dependent on device-specific empirical relations. Multidevice correlations of edge transport properties and edge transport barrier width would be a step in the right direction, but the ultimate goal must be the development and validation of theoretical models for the edge transport properties and the edge transport barrier width. The model should be of use in the near-term as a framework for the correlation of edge pedestal properties and, ultimately, for the prediction of pedestal properties in future devices.

TABLE V. Effect of auxiliary heating on the characteristic scale lengths in the edge transport barrier ($R=1.76$ m, $a=0.6$ m, $\kappa=1.76$, $\delta=0.22$, $B=2.0$ T, $I=1.0$ MA, $q_{95}=4.8$, $H_{89P}=2.0$, $H_N/H_{89}=0.5$, USN divertor, $S=3.0\times10^{21}$ s, $v_p=0.0$, $\chi_i=\chi_e=0.5$ m^2/s, $D=1/3\chi$, $C_0=0.02$, $C_2=1.5$, $Q_{\perp e}/Q_{\perp}=0.5$).

P_{nbi} (MW)	$Q_\perp$ (10^4 W/s)	$\Gamma_\perp$ (m^2 s)	L_n (cm)	L_{T_e} (cm)	L_{T_i} (cm)	L_{RD} (cm)	Δ_{TB} (cm)	BI
1.50	2.4	1.2	1.5	1.4	2.4	0.4	0.8	0.60
2.00	3.3	1.2	1.8	1.3	2.2	0.6	0.8	0.82
2.50	4.1	1.2	2.0	1.3	2.1	0.8	0.9	1.05
3.00	4.9	1.1	2.3	1.3	2.1	1.0	1.0	1.26

TABLE VI. Parameters for a typical case ($R=1.76$ m, $a=0.6$ m, $\kappa=1.76$, $\delta=0.22$, $B=2.0$ T, $I=1.0$ MA, $q_{95}=4.8$, $H_{89P}=2.0$, $H_N/H_{89}=0.5$, USN divertor, $S=12.0\times10^{21}$ s, $v_p=0.0$, $\chi_i=\chi_e=0.5$ m^2/s, $D=1/3\chi$, $C_0=0.02$, $C_1=1.0$, $C_2=1.5$, $Q_{\perp e}/Q_{\perp}=0.5$).

Plasma density (10^{20}/m^3)	
Divertor plate	1.20
Separatrix	0.12
Pedestal	0.23
Center	0.38
Plasma temperature (eV)	
Divertor plate	4
Separatrix electron/ion	73/109
Pedestal electron/ion	122/183
Center	3,860
Neutral concentration in transport barrier (%)	0.76
Heat flux through edge (10^4 W/m^2 s)	3.0
Particle flux through edge (10^{20}/m^2 s)	2.3
Characteristic pedestal scale lengths (cm)	
L_n	1.4
L_{T_e}	1.8
L_{T_i}	3.4
L_{MHD}	0.8
Δ_{TB}	0.9

APPENDIX: CALCULATION MODEL

1. Plasma core calculation (Refs. 9 and 10)

The average temperature in the core plasma is determined by equating the net heating (external heating less core impurity and bremsstrahlung radiation) to the power flux from the core into the SOL and then relating the power outflux to the average temperature and density and the energy confinement time. The core radiation is calculated by integrating a coronal equilibrium radiative transition calculation over a core ''parabola-to-a-power-on-a-pedestal'' profile ($x(r)=[x_0-x_{\mathrm{ped}}][1-(r/a)^2]^{\alpha}+x_{\mathrm{ped}}$) defined by input profile parameters ($\alpha_T=$parabola power coefficient, T_0/T_{ped} $=$center/pedestal temperature ratio). A ''noncoronal'' radiation enhancement factor may be input. The energy confinement time is calculated from the ITER89P scaling law with an input H_{89} enhancement factor.

The average core plasma density is determined by equating the total core fueling by neutral influx from the SOL and pellet and neutral beam fueling to the ion outflux into the SOL and then relating the ion outflux to the average ion density and the particle confinement time. Since recycling of neutrals is treated explicitly, the ion particle confinement time is taken from the scaling developed from measurement[11] of density die-away after pellet injection in DIII-D ($\tau_n=H_n\times0.51I^2$[MA]), rather than from an experimentally inferred particle confinement time that includes recycling neutrals. In this paper we assume that the same mechanisms affect energy and particle confinement by taking $H_n=H_{89}$. A ''parabola-to-a-power-on-a-pedestal'' density profile with input profile parameters (α_n and n/n_{ped}) and pedestal parameter ($n_{\mathrm{ped}}/\bar{n}$) is used to represent the core plasma density distribution, with the input parameters taken from experiment.

2. SOL and divertor plasma calculation (Refs. 9 and 10)

A ''two-point'' model of the SOL (scrape-off layer) and divertor plasma is obtained by integrating the density, momentum, and power balance equations over the length of the SOL and divertor channels to provide a calculation of the plasma temperature and density on the separatrix along the SOL region bounding the core plasma and on the separatrix in the recycling region just in front of the divertor target. Temperature and density in the divertor channel are determined by interpolation. Coronal equilibrium impurity radiation is included in the energy balance, with an input enhancement factor to account for noncoronal effects. The plasma balance equations contain terms to represent charge exchange, elastic scattering, and ionization. Volumetric recombination is represented in the recycling region. The width of the SOL is calculated from radial heat conduction, assuming Bohm transport, and a flux expansion factor taken from experiment is used to determine the width of the divertor regions. The heat and particle fluxes into the SOL from the core plasma calculation are inputs to the SOL and divertor plasma calculation. Standard sheath conditions at the divertor target are used. The calculation is made for the outer SOL and divertor leg.

3. Neutral transport calculation (Refs. 12 and 10)

The transport of neutral particles introduced by gas fueling, by volumetric recombination, and by recycling from the divertor plate and the chamber wall is modeled in the recycling regions, in the divertor channel regions, in the private flux regions, in the plenum regions, and in the scrape-off layer plenum region using the two-dimensional TEP (transmission/escape probabilities) method[13] to calculate inward fluxes of neutral particles into the scrape-off layer at the X point and at the ''midplane.'' These inward fluxes are then transported across the SOL and into the plasma core using the one-dimensional interface current balance method.[14] Neutrals (and ions) striking a material wall are reflected isotropically as atoms with probability R_N with one-half their incident energy and with probability $(1-R_N)$ as molecules that dissociate immediately to provide neutral atoms and ions with energy 2 eV. The atomic and molecular data and reflection coefficients are discussed in Refs. 15 and 16.

The plasma calculations described in Secs. I and II provide the background plasma in the plasma core, divertor regions, and SOL, and provide the neutral recycling and volumetric recombination sources for the neutral particle transport calculation. The neutral transport calculation in turn provides the fueling rate for the plasma core calculation and the atomic cooling and momentum damping rates and the ionization source for the plasma calculation in the SOL and divertor. The presence of plasma in the plenum and private flux regions is taken into account by assuming that neutral fluxes incident to these regions become isotropically distributed by charge-exchange and elastic scattering with plasma ions.

This modeling of neutral particle transport has been found to agree rather well with experiment and with Monte Carlo calculations.[16]

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